**Appendix A**

This appendix reviews the posterior sampling algorithm. The sampling that we present is direct application of DLM theory (West and Harrison 1997) and Gibbs sampling (Gelman et al. 2003). Our model is a type of linear state space models with individual level parameters in which we treat component of transition  and variance of components, as parameters to be estimated along with the sequence of state vectors () over time. Recall that we also explain individual level transition parameters with hyperparameters with components of () and variance of component () that we estimate along with other parameters. We begin with forward filtering step with the most recent values of:

**A.1. Sampling from** 

**Forward Filtering**. We use the standard DLM framework (Equation A1 to A5) to infer the posterior distribution  over time, where includes all information available to researcher at time t. We derive posterior  using multivariate normal theory.

Posterior distribution for,

 (A1)

Prior distribution for, where

, and. (A2)

Prior one-step-ahead forecast distribution: ,

,  (A3)

, with

 (A4)

Posterior distribution for, by marginal properties of normal distribution in (A4),

,, where  (A5)

**Backward Smoothing**. We derive backward smoothing algorithm by using (A1) and (A2) to get joint distribution of parameters at t and t-1, given information of previous moment  to obtain equation (A6):

. (A6)

Then, multivariate normal theory gives us the conditionals:

, (A7)

where and  is random draw from posterior 

We can conduct retrospective analysis using results of A6 and A7. In other word, we derive expectations and variance over all possible draws of posterior. As a result, our backward smoothing algorithm is:

 (A8)

And

. (A9)

**Simulation for**.

For:

Step 1. For compute the moments for the multivariate normal by sequential updating procedure described in forward filtering section above (Equations A2 - A5).

Step 2. At the end of the series, sample from the posterior distribution: 

Step 3. For sample  conditional on the latest draw.

The outcome of each of the iterations is the draws from the full conditional posterior.

**A.2. Sampling from** 

Conditional on all the states and the data, our DLM equation (1) and (2) simplify to a linear multivariate system with individual level hyperparameters with unknown parameters, and variance components . Consequently, the Gibbs sampler step with conjugate prior to estimate the joint posterior of the non-state parameters is as follows:

For:

1. We consider prior distribution from equation (3).
2. We assume error terms of the observation equation (1) and system Equation (2) are mutually independent. Thus, conditional on, we sample  independently.
3. Similarly, conditional on, we sampleseparately.

The outcome is the draws from full conditional posterior.

**A.3. Sampling from** 

Conditional on all the non-state parameters and the data, our equation (3-5) simplifies to linear multivariate system with unknown parameters and variance components. Therefore, the Gibbs sampler step with conjugate prior to estimate the joint posterior of the parameters is as follows:

For:

1. We consider prior distribution, vague prior.
2. Conditional on, we sample  independently.
3. Similarly, conditional on, we sampleseparately.